HIRDLS

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HIGH RESOLUTION DYNAMICS LIMB SOUNDER

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1. Elevation angle command correction
2. Dynamics equation of motion of scan mirror
3. Precompensation and elevation torque command correction
4. Optical bench inertial motion sensing and retrieval

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EOS

CONTENTS

1 SUMMARY

In this TC, the algorithms used to control the scan mirror during the uniform part of an elevation scan are derived. These algorithms include precompensation, feedforward compensation and feedback compensation. For the sake of the simplicity of the scanner control system, it is critical that the scan mirror be both statically and dynamically balanced. In this case, only the pitch and roll motions of the optical bench need to be measured to inertially stabilize the line of sight (LOS), and the compensation algorithms do not have to be changed depending on the gravity environment.

For reference, the algorithms for the control of a balanced scan mirror are summarized in the remainder of this section.

1.1 Notations

m_i	Output signal of accelerometer number *i* normalized by accelerometer scale factor
α_1	Actual optical bench inertial roll acceleration
α_2	Actual optical bench inertial pitch acceleration
$\hat{a}_1$	Estimate of α_1 retrieved from optical bench acceleration measurements
$\hat{a}_2$	Estimate of α_2 retrieved from optical bench acceleration measurements
$\hat{\alpha}_{1f}$	Filtered estimate of α_1
$\hat{\alpha}_{2f}$	Filtered estimate of α_2
θ_3	Actual optical bench inertial roll angle
θ_2	Actual optical bench inertial pitch angle
$\hat{\theta}_{3f}$	Filtered estimate of θ_3
$\hat{\theta}_{2f}$	Filtered estimate of θ_2
$\hat{\dot{\theta}}_{3f}$	Filtered estimate of $\dot{\theta}_3$
$\hat{\dot{\theta}}_{2f}$	Filtered estimate of $\dot{\theta}_2$
$\hat{\ddot{\theta}}_{3f}$	Filtered estimate of $\ddot{\theta}_3$
$\hat{\ddot{\theta}}_{2f}$	Filtered estimate of $\ddot{\theta}_2$
e	Actual scan mirror elevation shaft angle
e_0	elevation shaft angle when the scan mirror is in its unpowered equilibrium position
e_c	Commanded scan mirror shaft elevation angle
Δe_c	Scan mirror elevation angle command correction
$\Delta \dot{e}_c$	Scan mirror elevation rate command correction
$\Delta \ddot{e}_c$	Scan mirror elevation acceleration command correction
a	Actual scan mirror azimuth shaft angle
IS_{22}	Scan mirror inertia about elevation axis
K_s	Scan mirror flexure stiffness
D_s	Scanner elevation axis viscous damping coefficient
TRS	Scanner elevation motor torque

Tp	Precompensation component of scanner elevation motor torque
Tfb	Feedback component of scanner elevation motor torque
Tff	Feedforward component of scanner elevation motor torque

1.2 Control algorithms

In the following, all the algorithms are expressed in the continuous time domain. Which parts of these algorithms should be implemented digitally is a hardware implementation issue.

Retrieval of optical bench pitch and roll acceleration from accelerometer measurements

The current plan is to retrieve the inertial roll and pitch acceleration of the optical bench using four accelerometers at all times even though more accelerometers may be available for reliability and redundancy reasons.

$$\begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} = \begin{bmatrix} M_{a1} \\ M_{a2} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}$$

where $M_{\alpha 1}$ and $M_{\alpha 2}$ are constant matrices that depend on the accelerometer locations and measurement axes, or equivalently, if the accelerometer measurement axes are **all parallel** to one another:

$$\begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = \begin{bmatrix} D_{\alpha 1} \\ D_{\alpha 2} \end{bmatrix} \begin{bmatrix} m_1 - m_2 \\ m_1 - m_3 \\ m_1 - m_4 \\ m_2 - m_3 \\ m_2 - m_4 \\ m_3 - m_4 \end{bmatrix}$$

where $D_{\alpha 1}$ and $D_{\alpha 2}$ are constant matrices that depend on the accelerometer locations and the common accelerometer measurement axis. In this last case, the retrieval algorithm is insensitive to gravity.

Filtered estimates of optical bench pitch and roll motions

$$\begin{cases} \tilde{\ddot{\theta}}_{3f} = B(s)\hat{\alpha}_1 \\ \tilde{\ddot{\theta}}_{2f} = B(s)\hat{\alpha}_2 \end{cases}$$

where B(s) is the fourth-order bandpass Butterworth filter:

$$B(s)=\frac{s^2\left(\omega_1^2+\omega_2^2\right)}{\left(s^2+\sqrt{2}\omega_1 s+\omega_1^2\right)\left(s^2+\sqrt{2}\omega_2 s+\omega_2^2\right)}$$

where ω_1 and ω_2 are the lower and upper filter cut-off frequencies respectively.

$$\begin{cases}\dot{\hat{\theta}}_{3f}=\dfrac{1}{s}\ddot{\hat{\theta}}_{3f}\\ \dot{\hat{\theta}}_{2f}=\dfrac{1}{s}\ddot{\hat{\theta}}_{2f}\end{cases}$$

$$\begin{cases}\hat{\theta}_{3f}=\dfrac{1}{s}\dot{\hat{\theta}}_{3f}\\ \hat{\theta}_{2f}=\dfrac{1}{s}\dot{\hat{\theta}}_{2f}\end{cases}$$

Scan mirror shaft elevation angle, rate, and acceleration command corrections

$$\Delta e_c=\frac{-\cos(2\alpha)\hat{\theta}_{2f}+\sin(2\alpha)\theta_{3f}}{2\cos\alpha}$$

$$\Delta\dot{e}_c=\frac{-\cos(2\alpha)\dot{\hat{\theta}}_{2f}+\sin(2\alpha)\dot{\hat{\theta}}_{3f}}{2\cos\alpha}$$

$$\Delta\ddot{e}_c=\frac{-\cos(2\alpha)\ddot{\hat{\theta}}_{2f}+\sin(2\alpha)\ddot{\hat{\theta}}_{3f}}{2\cos\alpha}$$

Precompensation torque

$$Tp=IS_{22}\ddot{e}_c+D_s\dot{e}_c+K_s\left(e_c-e_0\right)$$

Feedforward correction torque

$$Tff=IS_{22}\Delta\ddot{e}_c+D_s\Delta\dot{e}_c+K_s\Delta e_c-IS_{22}\sin\alpha\ \hat{\alpha}_{1f}+IS_{22}\cos\alpha\ \hat{\alpha}_{2f}$$

Feedback torque

$$Tfb=K(s)\left(e_c+\Delta e_c-e\right)$$

where K(s) is the transfer function matrix of the PID controller

$$K(s)=\frac{K_i}{s}+K_p+K_d\frac{s}{1+s/\omega_d}$$

where K_i, K_p, K_d, ω_d are constants.

Control torque

$$TRS = Tp + Tff + Tfb$$

2 ELEVATION ANGLE COMMAND CORRECTION

The scan mirror elevation feedback control system regulates the angular position of the scan mirror relative to the optical bench. To meet the ITS rate and jitter requirements, however, we must control the angular position of the scan mirror relative to inertial space instead. In the presence of inertial optical bench motions, the precomputed scan mirror LOS elevation commands must therefore be corrected to account for these motions so that the desired inertial LOS motions are obtained. The object of this section is to derive the appropriate angle command corrections.

2.1 Preliminaries and assumptions

The optical bench is assumed rigid and is therefore represented by a single rigid body called O. Similarly, the scan mirror is represented by a single rigid body called S. It is assumed that all the components of the optical system at the exception of the scan mirror are rigidly connected to the optical bench. The telescope projected optical axis is then a line fixed at all times in the optical bench. The scan mirror datum position is used as a reference to define the scan mirror elevation and azimuth shaft angles. When the scanner is in its datum position, misalignments of the scanner azimuth and elevation axes, relative to the TRCF Z- and Y-axes, respectively, are negligible within the tolerance requirements given in the ITS. It is therefore assumed that the scanner azimuth and elevation axes are parallel to the TRCF Z- and Y-axes respectively when the scan mirror is in its datum position. Similarly, lack of orthogonality between the scan mirror elevation and azimuth axes is negligible, within the tolerance requirements given in the ITS, and it is therefore assumed that the scanner azimuth and elevation axes are orthogonal to one another. Finally, it is assumed that the scanner elevation and azimuth axes intersect at a point called the scanner datum point SD.

2.2 Nomenclature

$\{n_1, n_2, n_3\}$	Dextral set of orthogonal unit vectors fixed in a Newtonian reference frame N
$\{o_1, o_2, o_3\}$	Dextral set of orthogonal unit vectors fixed in the rigid body O (optical bench) and nominally parallel to $\{n_1, n_2, n_3\}$ $\{o_1, o_2, o_3\}$ can be regarded as the telescope reference coordinate frame (TRCF)
$\{s_1, s_2, s_3\}$	Dextral set of orthogonal unit vectors fixed in the rigid body S (scan mirror) and nominally parallel to $\{n_1, n_2, n_3\}$
$\theta_1, \theta_2, \theta_3$	Body 3_321 Euler angles describing the orientation of optical bench O in reference frame N $\theta_1, \theta_2, \theta_3$ are also called the optical bench yaw, pitch, and roll angles respectively
$\hat{r}$	Unit vector fixed in $\{o_1, o_2, o_3\}$ and parallel to the telescope projected optical axis
$\hat{i}$	Unit vector parallel to the instantaneous line of sight
E_0	Angle between o_1 and the telescope projected optical axis
E	Actual line-of-sight elevation angle

E_c	Commanded line-of-sight elevation angle
A	Actual line-of-sight azimuth angle
e	Actual scan mirror elevation shaft angle
a	Actual scan mirror azimuth shaft angle

2.3 Line-of-sight elevation angle

Following the definitions of the LOS azimuth and elevation angles given in the ITS, we have:

$$\hat{\mathrm{i}} = -\cos A \;\cos E \;\; n_1 - \sin A \;\cos E \;\; n_2 + \sin E \;\; n_3 \tag{1}$$

The unit vector defining the direction of the telescope projected optical axis is given by:

$$\hat{r} = \cos E_0 \;\; o_1 + \sin E_0 \;\; o_3 \tag{2}$$

A ray parallel to the telescope projected optical axis hitting the scan mirror is reflected out into the atmosphere into a ray parallel to the line of sight. We therefore have:

$$\hat{\mathrm{i}} = \hat{\mathrm{r}} - 2\left(\hat{\mathrm{r}} \cdot \mathrm{s}_1\right)\mathrm{s}_1 \tag{3}$$

Comparing the n_3 components of $\hat{\mathrm{i}}$ in equations (1) and (3), we find that:

$$\begin{aligned}\sin E = &\left[\sin E_0 \cos(2e) + \cos E_0 \cos a \;\; \sin(2e)\right]\cos q_2 \cos q_3 \\ &-\left[\cos E_0 + 2\left(\sin E_0 \sin e - \cos E_0 \cos a \;\; \cos e\right)\cos a \;\; \cos e\right]\sin q_2 \\ &+2\left(\sin E_0 \sin e - \cos E_0 \cos a \;\; \cos e\right) \;\sin a \;\; \cos e \;\; \cos q_2 \sin q_3\end{aligned} \tag{4}$$

We will find it convenient to introduce the following notations:

$$\lambda = \sqrt{\cos^2 \mathrm{E}_0 \;\cos^2 \mathrm{a} + \sin^2 \mathrm{E}_0} \tag{7}$$

$$\cos\phi = \frac{\cos \mathrm{E}_0 \cos \;\mathrm{a}}{\sqrt{\cos^2 \mathrm{E}_0 \cos^2 \mathrm{a} + \sin^2 \mathrm{E}_0}} \tag{8}$$

$$\sin\phi = \frac{\sin \mathrm{E}_0}{\sqrt{\cos^2 \mathrm{E}_0 \cos^2 \mathrm{a} + \sin^2 \mathrm{E}_0}} \tag{9}$$

Equation (6) can then be rewritten:

$$\begin{aligned}\frac{\sin E}{l} = &\sin(2e + f) \;\cos q_2 \cos q_3 - \left[\cos f \frac{\sin^2 a}{\cos a} - \cos a \;\; \cos(2e + f)\right]\sin q_2 \\ &- \sin a \;\left[\cos(2e + f) + \cos f\,\right] \;\cos q_2 \sin q_3\end{aligned} \tag{11}$$

Equation (11) gives the LOS elevation angle in terms of the scanner azimuth and elevation angles, and of the orientation of the optical bench relative to inertial space. It should be noted that the LOS elevation angle does not depend on the orientation of the optical bench in yaw.

2.4 Exact elevation angle command correction

Instead of working in both LOS and shaft elevation angle space, we will find it more convenient to work in shaft elevation angle space only. To this end, we introduce e_c according to the following equation:

$$\sin E_c = \lambda \sin(2e_c + \phi) \tag{12}$$

e_c is the scan mirror elevation shaft angle command that must be followed in order to obtain the desired LOS elevation angle E_c in the absence of inertial optical bench motion.

In the presence of inertial optical bench motion, the LOS elevation angle is equal to the commanded LOS elevation angle if and only if the scanner elevation angle e is equal to $\bar{e}_c$ which is defined by the following equation:

$$\begin{aligned} \sin(2e_c + \phi) = {} & \sin(2\bar{e}_c + \phi)\cos\theta_2 \cos\theta_3 - \left[\cos\phi \frac{\sin^2\alpha}{\cos\alpha} - \cos\alpha \ \cos(2\bar{e}_c + \phi)\right]\sin\theta_2 \\ & - \sin\alpha \ \left[\cos(2\bar{e}_c + \phi) + \cos\phi\right] \cos\theta_2 \sin\theta_3 \end{aligned} \tag{13}$$

$\bar{e}_c$ and not e_c should therefore be the scan mirror elevation shaft angle command in this case. Equation (13) can be solved explicitly for $\bar{e}_c$. The elevation angle command correction Δe_c is then given by:

$$\Delta e_c = \bar{e}_c - e_c \tag{14}$$

For the implementation of the LOS feedforward algorithms, however, we need only linear approximations to Δe_c . These approximations are given in the next section.

2.5 Linearized elevation angle command correction

To first order in θ_2, θ_3, and Δe_c, the elevation angle command correction is:

$$\Delta e_c = \frac{\cos\phi \sin^2\alpha - \cos^2\alpha \cos(2e_c + \phi)}{2\cos\alpha \ \cos(2e_c + \phi)}\theta_2 + \frac{\sin\alpha \ \left(\cos(2e_c + \phi) + \cos\phi\right)}{2\cos(2e_c + \phi)}\theta_3 \tag{15}$$

In the HIRDLS operation, the commanded elevation shaft angle will be on the order of a few degrees. The elevation angle command correction could therefore be further linearized with respect to e_c. When this is done, the expression of the elevation angle command correction reduces to:

$$\Delta e_c = \frac{-\cos(2a)\theta_2 + \sin(2a)\theta_3}{2\cos a} \tag{16}$$

The results of simulation studies not described in this TC show that (16) is likely to be an adequate approximation to the required elevation angle command correction, but of course better results are obtained if the approximation given in (15) is used instead.

3 DYNAMICS EQUATION OF MOTION OF SCAN MIRROR

In the following section, the dynamics equation of motion of the scan mirror is given when the scan mirror is maintained at **a fixed azimuth** angle and when the motion of the optical bench is **prescribed**. The scan mirror dynamics equation of motion derived under these conditions is appropriate to derive the torque component of the feedforward algorithms, because the scan mirror commanded accelerations are small during the uniform part of an elevation scan, and, because the inertia of the optical bench about the scan mirror elevation axis is about 4000 times larger than that of the scan mirror, the angular motion of the optical bench in first approximation is not significantly influenced by the scan mirror elevation motor torques reacted on the bench.

3.1 Preliminaries and assumptions

The optical bench is assumed rigid and is therefore represented by a single rigid body called O. Similarly, the scan mirror is represented by a single rigid body called S. It is assumed that the translational and rotational motions of the optical bench are not significantly affected by the motion of the scan mirror; these optical bench motions are therefore considered specified. It is assumed that the scan mirror azimuth angle is constant, since LOS feedforward stabilization is only needed under this condition. The scan mirror datum position is used as a reference to define the scan mirror elevation and azimuth shaft angles. The equilibrium position of the scan mirror on its flexural pivot in the absence of any motor torque need not coincide with the scan mirror datum position: that equilibrium position is defined by the elevation shaft angle e_0. When the scanner is in its datum position, misalignments of the scanner azimuth and elevation axes relative to the TRCF Z- and Y-axes, respectively, are negligible within the tolerance requirements given in the ITS. It is therefore assumed that the scanner azimuth and elevation axes are parallel to the TRCF Z- and Y-axes, respectively, when the scan mirror is in its datum position. Similarly, lack of orthogonality between the scan mirror elevation and azimuth axes within the tolerance requirements given in the ITS are negligible, and it is therefore assumed that the scanner azimuth and elevation axes are orthogonal to one another. Finally, it is assumed that the scanner elevation and azimuth axes intersect at a point called the scanner datum point, SD.

3.2 Nomenclature

$\{n_1, n_2, n_3\}$	Dextral set of orthogonal unit vectors fixed in a Newtonian reference frame N
$\{o_1, o_2, o_3\}$	Dextral set of orthogonal unit vectors fixed in the rigid body O (optical bench) and nominally parallel to $\{n_1, n_2, n_3\}$
	$\{o_1, o_2, o_3\}$ can be regarded as the telescope reference coordinate frame (TRCF)
$\{s_1, s_2, s_3\}$	Dextral set of orthogonal unit vectors fixed in the rigid body S (scan mirror) and nominally parallel to $\{n_1, n_2, n_3\}$
SD	Scan mirror datum point
$\theta_1, \theta_2, \theta_3$	Body 3_321 Euler angles describing the orientation of optical bench O in reference frame N
	$\theta_1, \theta_2, \theta_3$ are also called the optical bench yaw, pitch, and roll angles, respectively
$\vec{\alpha}$	Angular acceleration of O in reference frame N
$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$	Vector of coordinates of $\vec{\alpha}$ in reference frame $\{o_1, o_2, o_3\}$
$\vec{\gamma}$	Linear acceleration of SD in reference frame $\{n_1, n_2, n_3\}$
$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$	Vector of coordinates of $\vec{\gamma}$ in reference frame $\{n_1, n_2, n_3\}$
g	Acceleration of gravity
$\hat{G}$	Unit vector pointing in the direction of the gravity vector
$\begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}$	Vector of coordinates of $\hat{G}$ in reference frame $\{n_1, n_2, n_3\}$
SO	Scan mirror center of mass
SDSO	Position vector from SD to SO
$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$	Vector of coordinates of SDSO in reference frame $\{s_1, s_2, s_3\}$
M_s	Mass of scan mirror
e	Scan mirror shaft elevation angle
e_0	Elevation shaft angle when the scan mirror is in its equilibrium position
a	Scan mirror shaft azimuth angle
IS	Inertia dyadic of scan mirror S **about SD**

$\begin{bmatrix} IS_{11} & IS_{12} & IS_{13} \\ IS_{21} & IS_{22} & IS_{23} \\ IS_{31} & IS_{32} & IS_{33} \end{bmatrix}$	Matrix representation of dyadic IS in reference frame {s_1, s_2, s_3}, i.e., $IS = IS_{11}s_1s_1 + IS_{12}s_1s_2 + IS_{13}s_1s_3 + IS_{21}s_2s_1 + IS_{22}s_2s_2 + IS_{23}s_2s_3 + IS_{31}s_3s_1 + IS_{32}s_3s_2 + IS_{33}s_3s_3$
K_s	Scanner flexure stiffness
D_s	Scanner flexure viscous damping coefficient
TRS	Scanner elevation motor torque

3.3 Linearized dynamics equation of motion of scan mirror

The dynamics equation of motion of the scan mirror linearized with respect to θ_1, θ_2, θ_3, α_1, α_2, α_3, γ_1, γ_2, and γ_3 is:

$$IS_{22}\ddot{e} + D_s\dot{e} + K_s(e - e_0) = TRS$$

$$-M_s g\left[(l_1 \sin e - l_3 \cos e)\cos a \ G_1 + (l_1 \sin e - l_3 \cos e)\sin a \ G_2 + (l_1 \cos e + l_3 \sin e) \ G_3\right] \qquad \text{(a)}$$

$$+M_s g\left[(l_1 \sin e - l_3 \cos e)\sin a \ G_1 - (l_1 \sin e - l_3 \cos e)\cos a \ G_2\right]q_1 \qquad \text{(b)}$$

$$-M_s g\left[(l_1 \cos e + l_3 \sin e) \ G_1 - (l_1 \sin e - l_3 \cos e)\cos a \ G_3\right]q_2 \qquad \text{(c)}$$

$$+M_s g\left[(l_1 \cos e + l_3 \sin e) \ G_2 - (l_1 \sin e - l_3 \cos e)\sin a \ G_3\right]q_3 \qquad \text{(d)}$$

$$+M_s(l_1 \sin e - l_3 \cos e)\cos a \ \gamma_1 \qquad \text{(e)} \quad (17)$$

$$+M_s(l_1 \sin e - l_3 \cos e)\sin a \ \gamma_2 \qquad \text{(f)}$$

$$+M_s(l_1 \cos e + l_3 \sin e)\gamma_3 \qquad \text{(g)}$$

$$+\left[IS_{22} \sin a - IS_{12} \cos a \cos e - IS_{23} \cos a \sin e\right]\alpha_1 \qquad \text{(h)}$$

$$-\left[IS_{22} \cos a + IS_{12} \sin a \cos e + IS_{23} \sin a \sin e\right]\alpha_2 \qquad \text{(i)}$$

$$+\left[IS_{22} \sin e - IS_{23} \cos e\right]\alpha_3 \qquad \text{(j)}$$

To stabilize the instrument LOS in the presence of optical bench linear and angular motions, the scan mirror torque command TRS must be such that it cancels the terms (a) through (j) in Eq. (17). It should be noted that, in general, TRS depends on motions of the optical bench **in all degrees of freedom**.

The terms (a) through (d) arise because the gravity force acting on the scan mirror has a moment about the scan mirror elevation axis. The terms (e) through (g) arise because inertial forces acting on the scan mirror have a moment about the elevation axis. The terms (h) through (j) arise because inertial torques acting on the scan mirror have a moment about the elevation axis.

When the scan mirror is statically balanced (i.e., $l_1^2 + l_3^2 = 0$), the moment about the elevation axis of the gravity and inertial forces acting on the scan mirror vanishes. When the scan mirror is dynamically balanced (i.e., $IS_{12} = 0$ and $IS_{23} = 0$), the (j) term in Eq. (17) vanishes. The motion of the scan mirror, to first order, is then unaffected by the yaw angular motion of the optical bench.

For the sake of the simplicity of the LOS feedforward algorithms and of the inertial motion sensing system, the feedforward torque needed to stabilize the instrument LOS should depend on motions of the optical bench in as few degrees of freedom as possible. From this point of view, it is critical that the scan mirror be both **statically** and **dynamically balanced**. When this is the case, the dynamics of the scan mirror only depend on the pitch and roll motions of the optical bench. Also, to the extent that the optical bench inertial motion sensing system is insensitive to gravity, the feedforward algorithms become independent of gravity which is critical to the traceability of the LOS stability performance observed in the laboratory to the on-orbit situation.

When the scan mirror is statically and dynamically balanced, the linearized equation of motion of the scan mirror reduces to:

$$IS_{22}\ddot{e} + D_s\dot{e} + K_s\left(e - e_0\right) = TRS + IS_{22}\sin a\;\alpha_1 - IS_{22}\cos a\;\alpha_2 \tag{18}$$

4. PRECOMPENSATION AND ELEVATION TORQUE COMMAND CORRECTION

The current plan is to use a combination of precompensation, feedback compensation and feedforward compensation to control the scan mirror. Precompensation is an open-loop control strategy which is used to improve command following and settling time over what is achievable using feedback compensation alone. Accordingly, we decompose the scan mirror elevation torque TRS into three components: the precompensation component Tp, the feedforward correction component Tff, and the feedback component Tfb.

$$TRS = Tp + Tff + Tfb \tag{19}$$

The object of this section is to give the definitions and to derive the algorithms associated with each of these three torque components. We shall assume that the scan mirror is both statically and dynamically balanced.

4.1 Scan mirror precompensation torque

The scan mirror precompensation torque is the torque that would have to be applied to the scan mirror to exactly follow the desired LOS elevation angle E_c in the absence of inertial optical bench motion, assuming that the motion of the scan mirror is governed exactly by the dynamics equation (18). This torque component is given as follows:

$$Tp = IS_{22}\ddot{e}_c + D_s\dot{e}_c + K\left(e_c - e_0\right) \tag{20}$$

4.2 Scan mirror feedforward correction torque

The feedforward torque is the correction that must be added to the precompensation torque to exactly follow the desired LOS elevation angle E_c in the presence of inertial optical bench motion assuming that the motion of the scan mirror is governed exactly by the dynamics equation (18). Introducing the elevation angle command correction Δe_c, the elevation rate command correction $\Delta\dot{e}_c$, and the elevation acceleration command correction $\Delta\ddot{e}_c$, the feedforward torque can formally be expressed as follows:

$$\mathrm{Tff} = \mathrm{IS}_{22}\Delta\ddot{e}_c + \mathrm{D}_s\Delta\dot{e}_c + \mathrm{K}_s\Delta e_c - \mathrm{IS}_{22}\sin\alpha\;\alpha_1 + \mathrm{IS}_{22}\cos\alpha\;\alpha_2 \tag{21}$$

It can be shown that:

$$\Delta\dot{e}_c = \frac{\mathrm{d}\Delta e_c}{\mathrm{dt}} \tag{22}$$

$$\Delta\ddot{e}_c = \frac{\mathrm{d}^2\Delta e_c}{\mathrm{dt}^2} \tag{23}$$

where d/dt denotes the time derivative and where Δe_c is given by Eq. (15) or Eq. (16), depending on what elevation angle command correction formulae is used. Hence when the simpler elevation angle command correction formulae (16) is used, Tff is given by (21) together with:

$$\begin{aligned}\Delta e_c &= \frac{-\cos(2\alpha)\theta_2 + \sin(2\alpha)\theta_3}{2\cos\alpha}\\ \Delta\dot{e}_c &= \frac{-\cos(2\alpha)\dot{\theta}_2 + \sin(2\alpha)\dot{\theta}_3}{2\cos\alpha}\\ \Delta\ddot{e}_c &= \frac{-\cos(2\alpha)\ddot{\theta}_2 + \sin(2\alpha)\ddot{\theta}_3}{2\cos\alpha}\end{aligned} \tag{24}$$

When the more complex elevation angle command correction formulae given in (15) is used, the expression of the feedforward correction torque involves the first and second order time derivatives of the elevation command e_c and is significantly more complicated.

4.3 Scan mirror feedback torque

Precompensation and feedforward compensation are open-loop control strategies. Their performance depends on the accuracy of the models and on the accuracy of the approximations that went into their derivation. Feedback compensation provides the means to correct for residual LOS control errors after precompensation and feedforward compensation are applied to the scan mirror. In the HIRDLS feedback system, the scan mirror elevation angle commands are compared to the scan mirror shaft elevation angle measured by the MicroE encoder, and the differences are used to generate the feedback torque commands that are applied to the scan mirror to correct for those differences. The HIRDLS scan mirror feedback control system is a simple PID controller with transfer function K given by:

$$K(s) = \frac{K_i}{s} + K_p + K_d \frac{s}{1 + s / w_d} \tag{25}$$

where s is the usual Laplace variable. The PID controller gains K_i, K_p, and K_d are calculated as follows:

$$\mathrm{tc} = \frac{\left(1 + \sqrt{2}\right)}{2\pi \mathrm{bw}} \tag{26}$$

$$\mathrm{k} = \mathrm{IS}_{22} \frac{\left(1 + \sqrt{2}\right)}{\mathrm{tc}^2} \tag{27}$$

$$\mathrm{K}_\mathrm{i} = \mathrm{k} \frac{\left(\sqrt{2} - 1\right)}{\mathrm{tc}} \tag{28}$$

$$\mathrm{K}_\mathrm{p} = \mathrm{k} \tag{29}$$

$$\mathrm{K}_\mathrm{d} = \mathrm{k} \;\; \mathrm{tc} \tag{30}$$

where bw is the desired controller bandwidth.

Following the notations of the above sections, the elevation angle command error ε is:

$$\varepsilon = \bar{\mathrm{e}}_\mathrm{c} - \mathrm{e} \tag{31}$$

The scan mirror feedback torque is therefore given by:

$$\mathrm{Tfb} = \mathrm{K}(\mathrm{s})\left(\bar{\mathrm{e}}_\mathrm{c} - \mathrm{e}\right) \tag{32}$$

5 OPTICAL BENCH INERTIAL MOTION SENSING AND RETRIEVAL

In the previous sections, we have determined that, in their simplest form, the feedforward LOS stabilization system only needs information on the inertial motions of the optical bench in the pitch and roll degrees of freedom. The current plan is to use accelerometers to measure these inertial motions. This section derives the algorithms involved in the retrieval of the inertial pitch and roll motions of the optical bench from the accelerometer measurements. Several issues are discussed, including influence of gravity on the measurements, number of accelerometers required, and accelerometer configuration.

5.1 Preliminaries and assumptions

The accelerometers are assumed to be force-balanced servo accelerometers, and the accelerometer measurements are assumed to be AC coupled. To facilitate the derivation of the bench inertial motion retrieval algorithms, it is assumed that all the accelerometers have the same noise characteristics. This assumption only affects the derivation of the retrieval algorithms, not their complexity.

The optical bench is assumed rigid and is therefore represented by a single rigid body called O. It is assumed that the scanner elevation and azimuth axes intersect at a point called the scanner datum point, SD.

The inertial motions, rates, and accelerations of the optical bench are assumed to be small compared to unity.

5.2 Nomenclature

$\{n_1, n_2, n_3\}$	Dextral set of orthogonal unit vectors fixed in a Newtonian reference frame N
$\{o_1, o_2, o_3\}$	Dextral set of orthogonal unit vectors fixed in the rigid body O (optical bench) and nominally parallel to $\{n_1, n_2, n_3\}$
	$\{o_1, o_2, o_3\}$ can be regarded as the Telescope Reference Coordinate Frame (TRCF)
$\theta_1, \theta_2, \theta_3$	Body 3_321 Euler angles describing the orientation of optical bench O in reference frame N
	$\theta_1, \theta_2, \theta_3$ are also called the optical bench yaw, pitch and roll angles respectively
$\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$	Estimate of $\theta_1, \theta_2, \theta_3$ respectively
$\vec{\omega}$	Angular velocity of O in reference frame N
$\vec{a}$	Angular acceleration of O in reference frame N
$\hat{\vec{\alpha}}$	Estimate of $\vec{\alpha}$
$\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{bmatrix}$	Vector of coordinates of $\vec{\alpha}$ in reference frame $\{o_1, o_2, o_3\}$
$\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3$	Estimate of $\alpha_1, \alpha_2, \alpha_3$ respectively
SD	Point of optical bench defining the location of scanner datum
SD^0	Point fixed in Newtonian frame N defining the nominal location of scanner datum SD
$\vec{\gamma}$	Linear acceleration of SD in reference frame $\{n_1, n_2, n_3\}$
$\hat{\vec{\gamma}}$	Estimate of $\vec{\gamma}$
g	Acceleration of gravity
$\hat{G}$	Unit vector pointing in the direction of the gravity vector
A_i	Point of optical bench defining the location of accelerometer number *i*
A_i^0	Point fixed in Newtonian frame N defining the nominal location of accelerometer number i
$\vec{p}_i$	Position vector from SD to A_i
$\vec{p}_i^{\,0}$	Position vector from SD^0 to A_i^0
$\vec{r}_{ij}$	Position vector from A_i to A_j
$\vec{r}_{ij}^{\,0}$	Position vector from A_i^0 to A_j^0
$\hat{u}_i^0$	Unit vector fixed in $\{n_1, n_2, n_3\}$ pointing in the nominal direction of the measurement axis of accelerometer number *i*
$\hat{u}_i$	Unit vector fixed in $\{o_1, o_2, o_3\}$ pointing in the direction of the measurement axis of accelerometer number *i*
m_i	Output signal of accelerometer number *i* normalized by accelerometer scale factor

5.3 Accelerometer measurements

According to the assumptions and the definitions given in the last two sections, the measurement of accelerometer number *i* is given by:

$$m_i = \hat{u}_i \cdot \vec{\gamma} + (\vec{p}_i \times \hat{u}_i) \cdot \vec{\alpha} + [\vec{\omega} \times (\vec{\omega} \times \vec{p}_i)] \cdot \hat{u}_i + g\hat{G} \cdot (\hat{u}_i - \hat{u}_i^0) \tag{33}$$

where $\vec{u} \times \vec{v}$ denotes the cross-product of vectors $\vec{u}$ and $\vec{v}$, and where $\vec{u} \cdot \vec{v}$ denotes the inner-product of vectors $\vec{u}$ and $\vec{v}$.

To first order in $\vec{\omega}$, the gyroscopic term is negligible. Also, for small variations in the inertial orientation of the optical bench, the change in the direction of the accelerometer measurement axis can be expressed as:

$$\hat{u}_i - \hat{u}_i^0 = \vec{\theta} \times \hat{u}_i^0 \tag{34}$$

where $\vec{\theta} = \theta_3 n_1 + \theta_2 n_2 + \theta_1 n_3$. To first order, the expression for the measurement of accelerometer number *i* can therefore be rewritten as:

$$m_i = \hat{u}_i^0 \cdot \vec{\gamma} + (\vec{p}_i^0 \times \hat{u}_i^0) \cdot \vec{\alpha} - g(\hat{G} \times \hat{u}_i^0) \cdot \vec{\theta} \tag{35}$$

The accelerometer measurements are linear combinations of the optical bench inertial linear acceleration, inertial angular acceleration, and inertial angular position. Equation (35) gives simple explicit expressions for the coefficients involved in these linear combinations. Note that the linear acceleration and the angular position coefficients are independent of the accelerometer position which is important when only inertial angular accelerations need to be measured.

For HIRDLS, we would like the optical bench inertial motion sensing system to be insensitive to gravity so that the results of the performance verification tests performed in the laboratory can be traced to actual performance on-orbit. Such a motion sensing system can be devised because we only need to measure the inertial rotational motions of the optical bench. Recall that this is only the case when the scan mirror is statically balanced. Should the scan mirror be statically imbalanced, we would have to measure at least one component of the linear acceleration. An optical bench motion sensing system insensitive to gravity could then be constructed, but only for some specific orientation of the HIRDLS instrument in the 1g field. When only rotational motions need to be measured, one way to make the inertial motion sensing system insensitive to gravity is to use pairs of accelerometers having the same measurement axis.

If two accelerometers *i* and *j* have the same measurement axis, then, to first order, the difference between their normalized output signals is:

$$m_i - m_j = (\vec{r}_{ij}^0 \times \hat{u}_i^0) \cdot \vec{\alpha} \tag{36}$$

Only two such pairs, that is three accelerometers, are needed to measure the inertial pitch and roll motions of the optical bench. Measurement sensitivity for a given pair of accelerometers is proportional to accelerometer separation.

Although insensitivity of the optical bench motion sensing system to gravity is desired, it does not have to be perfectly achieved because the HIRDLS application does not require measuring angular accelerations down to low frequencies. For a given amplitude periodic angular motion of the optical bench at the frequency *f*, the relative magnitude R between the gravity and the angular acceleration terms in the accelerometer measurement equation (35) is roughly comparable to:

$$R \approx \frac{g\left\|\hat{G}\times\hat{u}_i^0\right\|}{\left\|\vec{p}_i^0\times\hat{u}_i^0\right\|(2pf)^2} \tag{37}$$

When $\vec{p}_i^0$ is normal to $\hat{u}_i^0$, $\left\|\vec{p}_i^0\right\|$ = 0.25 meter, and when $\hat{u}_i^0$ points 30 degrees off the vertical, then R is less than 0.1 for all frequencies above approximately 2.2 Hz. Since it is not expected that we will have to inertially rate stabilize the HIRDLS LOS below about 5 Hz, some degree of sensitivity of the optical bench angular motion sensing system to gravity can be tolerated.

5.4 Optical bench inertial angular acceleration retrieval algorithm

The current plan is to retrieve the inertial roll and pitch acceleration of the optical bench using four accelerometers at all times even though more accelerometers may be available for reliability and redundancy reasons. All accelerometers will have their measurement axis parallel to the TRCF Z-axis to maximize the sensitivity of the motion sensing system to pitch and roll motions (yaw accelerations are of no interest).

When all four accelerometers have the same noise characteristics, independently of whether the accelerometers have parallel measurement axes, **optimal** estimates of the optical bench inertial linear acceleration, inertial angular acceleration and inertial angular motion are obtained from the measurement equation (35) as follows:

$$\begin{bmatrix}\hat{\vec{\gamma}}\\ \hat{\vec{\alpha}}\\ \hat{\vec{\theta}}\end{bmatrix} = \begin{bmatrix}\left(\hat{u}_1^0\right)^T & \left(\vec{p}_1^0\times\hat{u}_1^0\right)^T & -g\left(\hat{G}\times\hat{u}_1^0\right)^T\\ \left(\hat{u}_2^0\right)^T & \left(\vec{p}_2^0\times\hat{u}_2^0\right)^T & -g\left(\hat{G}\times\hat{u}_2^0\right)^T\\ \left(\hat{u}_3^0\right)^T & \left(\vec{p}_3^0\times\hat{u}_3^0\right)^T & -g\left(\hat{G}\times\hat{u}_3^0\right)^T\\ \left(\hat{u}_4^0\right)^T & \left(\vec{p}_4^0\times\hat{u}_4^0\right)^T & -g\left(\hat{G}\times\hat{u}_4^0\right)^T\end{bmatrix}^{\#}\begin{bmatrix}m_1\\ m_2\\ m_3\\ m_4\end{bmatrix} \tag{38}$$

where $M^{\#}$ and M^T denotes the Moore-Penrose pseudo-inverse and the transpose of the matrix M respectively. Partitioning the pseudo-inverse of the regressor matrix involved in Eq. (38) consistently with $\begin{bmatrix}\hat{\vec{\gamma}}^T & \hat{\vec{\alpha}}^T & \hat{\vec{\theta}}^T\end{bmatrix}^T$ into $\begin{bmatrix}M_\gamma^T & M_\alpha^T & M_\theta^T\end{bmatrix}^T$, we arrive at:

$$\hat{\vec{\alpha}} = M_\alpha \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} \tag{39}$$

When the accelerometer measurement axes are all parallel to one another, it can be shown that (39) is equivalent to:

$$\hat{\vec{\alpha}} = \begin{bmatrix} \left(\vec{r}_{12}^{\,0} \times \hat{u}^0\right)^T \\ \left(\vec{r}_{13}^{\,0} \times \hat{u}^0\right)^T \\ \left(\vec{r}_{14}^{\,0} \times \hat{u}^0\right)^T \\ \left(\vec{r}_{23}^{\,0} \times \hat{u}^0\right)^T \\ \left(\vec{r}_{24}^{\,0} \times \hat{u}^0\right)^T \\ \left(\vec{r}_{34}^{\,0} \times \hat{u}^0\right)^T \end{bmatrix}^{\#} \begin{bmatrix} m_1 - m_2 \\ m_1 - m_3 \\ m_1 - m_4 \\ m_2 - m_3 \\ m_2 - m_4 \\ m_3 - m_4 \end{bmatrix} \tag{40}$$

where $\hat{u}^0$ denotes the common accelerometer measurement axis. Upon Eq. (36), the above equivalence shows that the optimal retrieval algorithms are independent of gravity in this case. This is true for any choice of the common accelerometer measurement axis.

The retrieval algorithms given in (39) or (40) can easily be generalized to the case where more than four accelerometers are used. To obtain the correct result in (40), all possible unordered accelerometer pairs (i.e., $n(n\text{-}1)/2$ pairs where n is the number of accelerometers) must be included.

5.5 Retrieval of feedforward component of optical bench inertial pitch and roll angles and accelerations

Filtered estimates rather than the raw estimates of the inertial pitch and roll angles and accelerations will be used in the LOS feedforward algorithms. The raw estimates of the roll and pitch accelerations are given by:

$$\begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = \begin{bmatrix} M_{\alpha 1} \\ M_{\alpha 2} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} \tag{41}$$

where $M_{\alpha 1}$ and $M_{\alpha 2}$ are the first and second rows of M_α respectively, if Eq. (39) is used, or equivalently by:

$$\begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = \begin{bmatrix} D_{\alpha 1} \\ D_{\alpha 2} \end{bmatrix} \begin{bmatrix} m_1 - m_2 \\ m_1 - m_3 \\ m_1 - m_4 \\ m_2 - m_3 \\ m_2 - m_4 \\ m_3 - m_4 \end{bmatrix} \tag{42}$$

where $D_{\alpha 1}$ and $D_{\alpha 2}$ are the first and second rows respectively of the matrix pseudo-inverse in (40), if Eq. (40) is used and if the accelerometer measurement axes are all parallel to one another.

LOS feedforward stabilization for HIRDLS will be approximately limited to the 5 to 100 Hz frequency range. Below about 5 Hz (TBV), inertial stabilization is not required and is limited by accelerometer drift. Above 100 Hz, inertial stabilization is limited by optical bench flexibility. To avoid adding a drift term on the scanner elevation command at low frequencies and to avoid feeding optical bench structural vibrations to the scanner motor at high frequencies, the raw estimates of the optical bench pitch and roll accelerations will be bandpass filtered before they are used in the feedforward algorithms. Fourth order Butterworth filters are representative of the bandpass filters required. Such filters are of the form:

$$B(s) = \frac{s^2\left(\omega_1^2 + \omega_2^2\right)}{\left(s^2 + \sqrt{2}\omega_1 s + \omega_1^2\right)\left(s^2 + \sqrt{2}\omega_2 s + \omega_2^2\right)} \tag{43}$$

where *s* is the usual Laplace variable, and where ω_1 and ω_2 are the lower and upper filter cut-off frequencies respectively. Typical filtered estimates of the roll and pitch accelerations are therefore of the form:

$$\begin{aligned} \ddot{\hat{\theta}}_{3f} &= B(s)\hat{\alpha}_1 \\ \ddot{\hat{\theta}}_{2f} &= B(s)\hat{\alpha}_2 \end{aligned} \tag{44}$$

Filtered estimates of the roll and pitch angles are obtained by integrating the filtered roll and pitch acceleration estimates:

$$\hat{\dot{\theta}}_{3f} = \frac{B(s)}{s}\hat{\alpha}_1$$
$$\hat{\dot{\theta}}_{2f} = \frac{B(s)}{s}\hat{\alpha}_2 \tag{45}$$

Filtered estimates of the roll and pitch angles are obtained by integrating the filtered roll and pitch acceleration estimates twice:

$$\hat{\theta}_{3f} = \frac{B(s)}{s^2}\hat{\alpha}_1$$
$$\hat{\theta}_{2f} = \frac{B(s)}{s^2}\hat{\alpha}_2 \tag{46}$$

5.6 Hardware implementation of motion sensing system

Equations (41) and (42) point to two different hardware implementations of the optical bench motion sensing system. According to the first equation, one would sample the raw accelerometer analog signals and digitally perform a matrix vector multiplication to get the estimate the optical bench angular acceleration. According to the second equation, one would first difference the accelerometer signals (using differential amplifiers for example), then sample the differences and finally digitally perform a matrix vector multiplication to get the estimate of the optical bench angular acceleration. The second implementation is more complex and becomes impractical when more than four accelerometers are used. For HIRDLS, however, it is the recommended one because we will likely have to measure small optical bench angular accelerations in the presence of relatively large optical bench linear accelerations. Also, in that second implementation, the resolution and range of the analog-to-digital converters can be tailored to the expected angular accelerations, which are the quantities of direct interest, rather than the relatively large maximum expected raw accelerometer signals. Reaching a final decision regarding the appropriate implementation may require some experimental testing.

When the accelerometer measurement axes are not exactly parallel to one another or when the accelerometers have slightly different noise characteristics, the retrieval algorithm given by Eq. (42) is not optimal and the estimates of the angular acceleration are affected by gravity at low frequencies. Some analysis will have to be performed to determine what the maximum variations in accelerometer noise characteristics and what the maximum degree of accelerometer measurement axis misalignment are that would allow us to use the retrieval algorithm given in Eq. (42) without significant loss of accuracy.

Bandpass filtering of the optical bench acceleration estimates could be done in analog, digitally, or a combination of both. All digital implementations are the simplest since only two signals need to be filtered. All analog implementations are the most complex since all analog signals must be filtered (i.e., 8 or 12 signals for a fully redundant sensing system depending on the version of retrieval algorithm used), but they provide good protection against aliasing. Implementing the low-pass portion of the filters in analog and the high-pass portion of the filters digitally may provide a suitable compromise between complexity and performance.